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Stability of tubular film flow: a model of the film-blowing process

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Methods of linear hydrodynamic stability are applied to analyse the stability of the isothermal Newtonian tubular film flow model of the industrial film-blowing process. The infinitesimal disturbances are assumed to possess axisymmetry. The relevant eigenvalue problem is formulated. A straightforward numerical scheme is developed to deal with the problem. Results are presented in the form of neutral-stability curves in the relevant parameter space. The significance and relevance of these to the industrial process are briefly discussed.

1. Introduction

In a series of papers Pearson & Petrie (1970*a*–*c*) analysed the film-blowing process, an industrial process for manufacturing thin polymer film. These authors developed a mathematical model of the kinematics and dynamics of the process. In the film-blowing process (see figure 1), polymer melt is extruded continuously through a circular slot die to form a thin-walled tube. This tube is drawn upwards by being passed through a pair of driven nip rolls. The nip rolls also form an air-tight seal. Air introduced through the centre of the die is retained inside the tube and inflates it into an elongated bubble. Thus as the polymer film moves towards the nip rolls it is being drawn longitudinally by these rolls and stretched transversely by the internal pressure. The transition of the molten polymer to a solid film is accelerated and localized by a jet of cold air directed onto the outer surface of the film from an annular ring just above the die. A fairly distinct freeze-line is formed. Above the freeze-line the film undergoes no further deformation. Banks of converging rollers are placed below the nip rolls to guide and collapse the bubble. For further details of the process see, for example, Schenkel (1966, p. 319).

The free-surface tubular film flow of Pearson & Petrie attempts to model the flow of the molten polymer between the die exit and the freeze-line. To keep the problem manageable these authors assumed the flow to be isothermal in the flow field of interest, and the material to be an incompressible Newtonian fluid with constant viscosity μ. Since the thickness of the film is everywhere small compared with other linear dimensions of the flow field, it has been shown that the variations of all flow variables across the thickness of the film can be neglected compared with their variations in the direction of the flow. In normal film-blowing operation the viscosity of the melt is high and the velocity involved is small, so

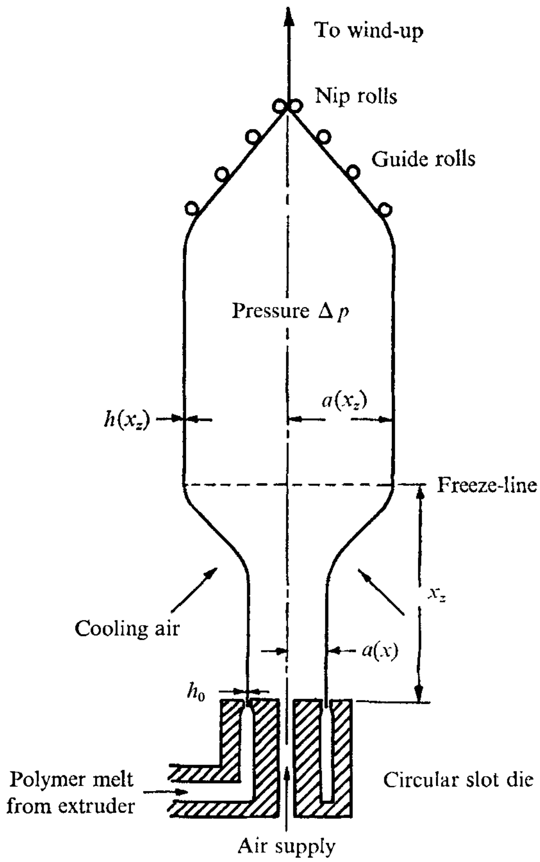


FIGURE 1. The film-blowing process (after Pearson & Petrie 1970*a*).

that the appropriate Reynolds number is small. Under these conditions viscous forces will dominate and all but viscous and pressure forces in the momentum equations can be neglected. The dependent variables in these equations are the radius of the tube a, the thickness of the film h and the velocity of the fluid v. See figure 2. These variables are functions of the axial co-ordinate of the cylindrical polar co-ordinate system (ρ, ϕ, x). The freeze-line height x_z is an independent parameter of the isothermal flow.

This paper examines the stability of the film-blowing process. Infinitesimal disturbances are superimposed on the steady Pearson & Petrie model. Methods of linear hydrodynamic stability theory are applied to study the temporal behaviour of these disturbances. Only axisymmetric disturbances are considered. For the more general problem of non-axisymmetric disturbances see Yeow (1972).

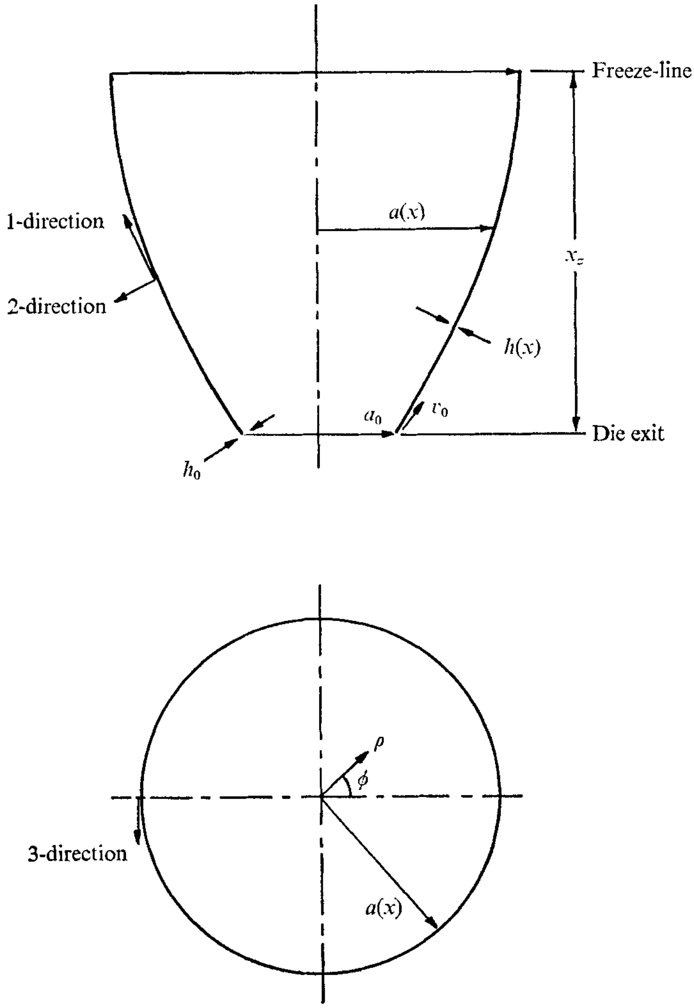


FIGURE 2. The tubular film flow model (after Pearson & Petrie 1970*a*).

2. The steady flow equations

This section summarizes the results of Pearson & Petrie needed in the stability analysis.

The rate-of-strain tensor at any point (ϕ, x) on the film can be shown to be

$$\left.\begin{aligned} e_{11} = \cos\theta\frac{dv}{dx}, \quad e_{22} = \frac{\cos\theta}{h}v\frac{dh}{dx}, \\ e_{33} = \frac{\cos\theta}{a}v\frac{da}{dx}, \end{aligned}\right\} \tag{1}$$

where θ is the angle between the bubble surface and the vertical, i.e.

$$\tan\theta = da/dx.$$

The 1-, 2- and 3-directions are respectively the direction of flow, the direction of the outward-pointing normal to the film and the azimuthal direction. The stress tensor is related to the rate-of-strain tensor by the simple Newtonian constitutive equation

$$t_{ij} = -p\delta_{ij} + 2\mu e_{ij}.$$

p is the isotropic pressure. The condition that the normal stress is zero (relative to atmospheric pressure) at the free surfaces gives

$$\left.\begin{aligned} t_{11} &= 2\mu\left(\frac{dv}{dx} - \frac{v}{h}\frac{dh}{dx}\right)\cos\theta, \\ t_{33} &= 2\mu v\left(\frac{1}{a}\frac{da}{dx} - \frac{1}{h}\frac{dh}{dx}\right)\cos\theta. \end{aligned}\right\} \tag{2}$$

The shape of the bubble is determined by the balance between the viscous stresses t_{11} and t_{33}, the axial tension f_z applied by the nip rolls and the slight constant positive pressure Δp inside the bubble. A force balance on the section of the bubble between the heights x and $x+dx$ gives

$$d(2\pi h t_{11} a \cos\theta)/dx = d(\pi a^2 \Delta p)/dx. \tag{3}$$

Integrating this between the height x and the freeze-line yields

$$2\pi a p_l \cos\theta + \pi(a_z^2 - a^2)\,\Delta p = f_z,$$

where p_l is the viscous force per unit width of the bubble in the 1-direction, i.e. $p_l = t_{11} h$. a_z is the radius at $x = x_z$. Treating the thin film as a membrane, force balance in the normal direction requires

$$\Delta p = p_l/r_1 + p_h/r_3, \tag{4}$$

where p_h is the viscous force per unit width of the bubble in the 3-direction, i.e. $p_h = t_{33} h$. r_1 and r_3 are the two principal radii of curvature of the surface, i.e. $r_1 = -\sec^3\theta(d^2a/dx^2)^{-1}$ and $r_3 = a\sec\theta\ (\equiv a/\cos\theta)$. A final relationship among a, h and v is

$$Q = 2\pi a h v,$$

where Q is the steady volumetric flow rate from the circular slot die, assuming $h \ll a$.

Introducing dimensionless variables

$$\left.\begin{aligned} A = a/a_0,\quad H = h/h_0,\quad X = x/a_0,\quad V = v/v_0, \\ T_{11} = t_{11}h_0/(\Delta p a_0),\quad T_{33} = t_{33}h_0/(\Delta p a_0) \end{aligned}\right\} \tag{5}$$

and dimensionless parameters

$$B = \pi a_0^3 \Delta p/(\mu Q),\quad T_z = a_0 f_z/(\mu Q),$$
$$X_z = x_z/a_0,\quad R = a_z/a_0,\quad T^* = T_z - R^2 B,$$

the above set of equations reduces to

$$A' = \tan\theta,$$
$$H'/H = -A'/2A - \tfrac{1}{4}\sec^2\theta(T^* + A^2 B),$$
$$2A^2(T^* + A^2B)\,\theta' = 3\sin^2\theta + A(T^* - 3A^2B).$$

A prime denotes d/dx, and a_0, h_0 and v_0 are the values of a, h and v at $X = 0$ respectively.

The set of boundary conditions relevant to the physical problem that motivated the analysis is

$$A = H = 1 \quad \text{at} \quad X = 0,$$
$$dA/dX = 0 \quad \text{at} \quad X = X_z.$$

Numerical solutions of these equations are given by Pearson & Petrie. These authors also discussed the nature of the solutions and the physical interpretations of the parameters B, T_z and X_z. Under normal operating conditions of the film-blowing process these parameters assume values in the ranges

$$0{\cdot}075 \leqslant B \leqslant 0{\cdot}4, \quad 0{\cdot}5 \leqslant T_z \leqslant 2{\cdot}5, \quad 5 \leqslant X_z \leqslant 20.$$

3. Axisymmetric disturbances

Methods of linear hydrodynamic stability are now applied to study the effect of small disturbances on the steady tubular film flow. In the perturbed flow it is assumed that Δp remains unchanged. To keep the analysis simple, disturbances are assumed to be axisymmetric and the velocity disturbance is assumed to have no component in the azimuthal direction. The disturbances are decomposed into Fourier components of the form

$$\left.\begin{aligned} A(X,T) &= \bar{A}(X)(1+a^*(X)\,e^{-i\Omega T}),\\ H(X,T) &= \bar{H}(X)(1+h^*(X)\,e^{-i\Omega T}),\\ V(X,T) &= \bar{V}(X)(1+v^*(X)\,e^{-i\Omega T}).\end{aligned}\right\} \tag{6}$$

Terms involving starred quantities are the time-dependent disturbances. These are assumed to be small compared with the steady flow variables, which are denoted by overbars. Two new dimensionless variables have been introduced in the above equations: a dimensionless time T and a dimensionless frequency Ω. These are defined by

$$T = t(v_0/a_0), \quad \Omega = \omega(a_0/v_0).$$

The physical interpretations of t and ω are self-evident.

Let (x^1, x^2, x^3) be a dimensionless cylindrical polar co-ordinate system, i.e. $x^1 = \rho/a_0$, $x^2 = \phi$ and $x^3 = X$. Superscripts are used here to signify the contravariant nature of these co-ordinates. Let (u^1, u^3) be the dimensionless surface co-ordinates defined on the inner surface of the bubble by $u^1 = x^3$ and $u^3 = x^2$. The metric tensor $\{a_{\alpha\beta}\}$ of this surface co-ordinate system is related to the spatial metric tensor $\{g_{ij}\}$ of the (x^1, x^2, x^3) co-ordinates by

$$a_{\alpha\beta} = g_{ij}t^i_\alpha t^j_\beta, \quad i,j = 1,2,3, \quad \alpha,\beta = 1,3,$$

where $t^i_\alpha = \partial x^i/\partial u^\alpha$. In the above equations summation over repeated indices is understood. Written explicitly the surface metric tensor is

$$\{a_{\alpha\beta}\} = \begin{bmatrix} 1+(\partial A/\partial X)^2 & 0 \\ 0 & A^2 \end{bmatrix}.$$

Based on the surface co-ordinates u^1 and u^3, a spatial co-ordinate system (u^1, u^2, u^3) is now defined in the region occupied by the film. u^2 is taken to be in the direction of the outward-pointing normal to the inner surface of the bubble such that $(ds)^2 = h^2(du^2)^2$, where ds denotes an infinitesimal displacement. For $\bar{H} \ll 1$, u^1, u^2 and u^3 form an orthogonal co-ordinate system. The metric tensor of this spatial co-ordinate system is

$$\{a_{ij}\} = \begin{bmatrix} 1+(\partial A/\partial X)^2 & 0 & 0 \\ 0 & H^2 & 0 \\ 0 & 0 & A^2 \end{bmatrix}.$$

Upon substituting (6) into the above and linearizing with respect to the infinitesimal disturbances, $\{a_{ij}\}$ simplifies to

$$\{a_{ij}\} = \begin{bmatrix} 1+\left(\dfrac{d\bar{A}}{dX}\right)^2 + 2\dfrac{d\bar{A}}{dX}\dfrac{d(\bar{A}a^*)}{dX} & 0 & 0 \\ 0 & \bar{H}^2+2\bar{H}^2h^* & 0 \\ 0 & 0 & \bar{A}^2+2\bar{A}^2a^* \end{bmatrix}.$$

For simplicity the exponential time factor $e^{-i\Omega T}$ has been omitted from the above. It is clear that this metric tensor is time dependent. Also, in this co-ordinate system the only non-vanishing velocity component is $V(X, T)$, in the 1-direction.

In a co-ordinate system with a time-dependent metric tensor, the covariant components of the rate-of-strain tensor are given by (Aris 1962, p. 228)

$$e_{ij} = \tfrac{1}{2}\partial a_{ij}/\partial t + \tfrac{1}{2}(v_{i,j} + v_{j,i}).$$

Commas between subscripts denote covariant differentiation. The time derivative in the above equation is the contribution to the rate of strain arising from the 'stretching' of the co-ordinates. Converting covariant components to physical components, the non-vanishing components of the rate-of-strain tensor become

$$E_{11} = \frac{1}{a_{11}^{\frac{1}{2}}}\frac{\partial a_{11}^{\frac{1}{2}}}{\partial T} + \frac{1}{a_{11}^{\frac{1}{2}}}\frac{\partial V}{\partial u^1},$$

$$E_{22} = \frac{1}{a_{22}^{\frac{1}{2}}}\frac{\partial a_{22}^{\frac{1}{2}}}{\partial T} + \frac{1}{a_{11}^{\frac{1}{2}}}\frac{\partial \ln a_{22}^{\frac{1}{2}}}{\partial u^1}V,$$

$$E_{33} = \frac{1}{a_{33}^{\frac{1}{2}}}\frac{\partial a_{33}^{\frac{1}{2}}}{\partial T} + \frac{1}{a_{11}^{\frac{1}{2}}}\frac{\partial \ln a_{33}^{\frac{1}{2}}}{\partial u^1}V.$$

Substituting the linearized metric tensor given above and V from (6), the rate-of-strain tensor of the disturbed flow reduces to

$$\begin{aligned} E_{11} &= \bar{E}_{11} + e_{11}^* \\ &= \frac{d\bar{V}}{dX}\cos\theta + \left(\cos\theta\frac{d(\bar{V}v^*)}{dX} - \bar{E}_{11}\sin\theta\cos\theta\frac{d(\bar{A}a^*)}{dX} - i\Omega\sin\theta\cos\theta\frac{d(\bar{A}a^*)}{dX}\right), \end{aligned} \tag{7a}$$

$$\begin{aligned} E_{22} &= \bar{E}_{22} + e_{22}^* \\ &= \frac{\bar{V}}{\bar{H}}\frac{d\bar{H}}{dX}\cos\theta + \left(\frac{\bar{V}}{\bar{H}}\frac{d(\bar{H}h^*)}{dX}\cos\theta + \bar{E}_{22}\left(v^* - h^* - \cos\theta\sin\theta\frac{d(\bar{A}a^*)}{dX}\right) - i\Omega h^*\right), \end{aligned} \tag{7b}$$

$$E_{33} = \bar{E}_{33} + e_{33}^*$$
$$= \frac{\bar{V}}{\bar{A}}\frac{d\bar{A}}{dX}\cos\theta + \left(\frac{\bar{V}}{\bar{A}}\frac{d(\bar{A}a^*)}{dX}\cos\theta + \bar{E}_{33}\left(v^* - a^* - \cos\theta\sin\theta\frac{d(\bar{A}a^*)}{dX}\right) - i\Omega a^*\right), \quad (7c)$$

where $\tan\theta = d\bar{A}/dX$. All quantities from here on are dimensionless and linearization is understood. e_{11}^*, e_{22}^* and e_{33}^* are the disturbances to the rate-of-strain tensor. This tensor has been made dimensionless with respect to a_0/v_0. In terms of these disturbances, the stress tensor becomes

$$T_{11} = \bar{T}_{11} + t_{11}^*, \quad T_{33} = \bar{T}_{33} + t_{33}^*,$$

where

$$t_{11}^* = (e_{11}^* - e_{22}^*)/B, \quad t_{33}^* = (e_{33}^* - e_{22}^*)/B. \quad (8)$$

The stress components of the steady flow, $\bar{T}_{11}$ and $\bar{T}_{33}$, are defined in (5), t_{11}^* and t_{33}^* are the disturbances suffered by $\bar{T}_{11}$ and $\bar{T}_{33}$. The parameter B arises from the way the stress and rate-of-strain tensors were made dimensionless.

All the disturbance quantities can be expressed ultimately in terms of a^*, h^* and v^*. Three equations have to be found for these three variables. The first of these is provided by the continuity equation for incompressible fluid:

$$e_{11}^* + e_{22}^* + e_{33}^* = 0. \quad (9)$$

From (4), the first-order terms give

$$(\bar{T}_{11}\bar{H}h^* + \bar{H}t_{11}^*)/\bar{R}_1 - \bar{H}\bar{T}_{11}r_1^*/\bar{R}_1^2 + (\bar{T}_{33}\bar{H}h^* + \bar{H}t_{33}^*)/\bar{R}_3 - \bar{H}\bar{T}_{33}r_3^*/\bar{R}_3^2 = 0. \quad (10)$$

This is the second of the three equations relating a^*, h^* and v^*. $\bar{R}_1$ and $\bar{R}_3$ are the dimensionless principal radii of curvature of the steady bubble; r_1^* and r_3^* are the disturbances to these radii:

$$\left.\begin{aligned} R_1 &= \bar{R}_1 + r_1^*, \quad R_3 = \bar{R}_3 + r_3^*, \\ \bar{R}_1 &= r_1/a_0 = -\sec^3\theta(d^2\bar{A}/dX^2)^{-1}, \\ \bar{R}_3 &= r_3/a_0 = \bar{A}\sec\theta, \\ r_1^* &= \bar{R}_1\left(3\sin\theta\cos\theta\frac{d(\bar{A}a^*)}{dX} - \frac{d^2(\bar{A}a^*)}{dX^2}\bigg/\frac{d^2\bar{A}}{dX^2}\right), \\ r_3^* &= \bar{R}_3(a^* + \cos\theta\sin\theta\, d(\bar{A}a^*)/dX). \end{aligned}\right\} \quad (11)$$

These equations are most easily derived by considering the principal radii of curvature in terms of ratios of the coefficients of the first and second fundamental forms of the surface. See, for example, Aris (1962, pp. 213 and 216).

The third equation is obtained by substituting (7) and (8) into the dimensionless equivalent of (3). The first-order terms give

$$\frac{d}{dX}\left(\bar{A}\bar{H}t_{11}^*\cos\theta + \bar{A}\bar{H}\bar{T}_{11}h^*\cos\theta + \bar{A}\bar{H}\bar{T}_{11}a^*\cos\theta - \bar{A}\bar{H}\bar{T}_{11}\frac{d\bar{A}a^*}{dX}\sin\theta\cos^2\theta\right) - \frac{d}{dX}(\bar{A}^2a^*) = 0. \quad (12)$$

Equations (7), (8) and (11) can be used to eliminate e_{11}^*, e_{22}^*, e_{33}^*, t_{11}^*, t_{33}^*, r_1^* and r_3^* from (9), (10) and (12) to give a set of three simultaneous linear homogeneous ordinary differential equations for a^*, h^* and v^*. It will be found that these equations contain first and second derivatives of a^* and h^* and the first derivative of v^*. In examining the stability of the tubular film flow to small disturbances, interest is focused on the temporal behaviour of disturbances with homogeneous boundary conditions. Five homogeneous boundary conditions have to be prescribed in the present case. Three of these are easily found:

$$a^*(0) = v^*(0) = h^*(0) = 0. \tag{13}$$

If a constant velocity is assumed for the winding up of the solid film then at the freeze-line the velocity perturbation must vanish, i.e.

$$v^*(X_z) = 0. \tag{14}$$

The final boundary condition is obtained from the physical assumption that once the bubble passes through the freeze-line its diameter cannot undergo any further changes (Pearson & Petri 1970c), i.e.

$$DA/DT = 0 \quad \text{at} \quad X = X_z,$$

where D/DT is the material derivative operator. In the present case, this boundary condition reduces to

$$\bar{V}\, d(\bar{A}a^*)/dX - i\Omega\bar{A}a^* = 0 \quad \text{at} \quad X = X_z. \tag{15}$$

The steady flow boundary condition $d\bar{A}/dX = 0$ at X_z has been used in the above derivation.

Equations (9), (10) and (12) together with the boundary conditions (13)–(15) constitute the eigenvalue problem relevant to the stability of the tubular film flow to axisymmetric disturbances. The parameters of the problem are B, T_z and X_z, which together characterize the steady flow, and Ω, the complex frequency of a particular Fourier component of the disturbance. B, T_z and X_z enter the problem through the steady solution $\bar{A}$, $\bar{H}$ and $\bar{V}$. Ω appears where time derivatives occur. For each point in B, T_z, X_z space it is expected that there exists a set of discrete complex $\Omega = \Omega_r + i\Omega_i$ for which the eigenvalue problem has non-trivial solutions. Of greatest interest is the element of this set which has the largest Ω_i. The locus of the points in B, T_z, X_z space for which max $\Omega_i = 0$ is the neutral-stability surface of the flow.

4. Numerical procedure

A straightforward numerical scheme has been developed to deal with the eigenvalue problem formulated in the previous section. This scheme attempts to locate the neutral-stability curve in the B, T_z plane for a few selected values of X_z. The amount of computation needed does not permit complete determination of the neutral-stability surface in the entire region of B, T_z, X_z space relevant to the film-blowing process.

Equation (12) can be integrated once and rearranged to give

$$t^*_{11} = -\{[\bar{A}\bar{H}\bar{T}_{11}(h^*\cos\theta + a^*\cos\theta - (d(\bar{A}a^*)/dX)\sin\theta\cos^2\theta) - \bar{A}^2a^*]^X_{X_z} - (\bar{A}\bar{H}t^*_{11}\cos\theta)_{X_z}\}/(\bar{A}\bar{H}\cos\theta)_X.$$

This equation expresses t^*_{11} at X as a function of a^*, h^*, v^* and da^*/dX at X and the values of these variables at the boundary $X = X_z$. This integration reduces the order of the differential system from five to four. The amount of computation is also reduced. Note that $t^*_{11}(X_z)$, introduced in the above equation, can be expressed in terms of boundary conditions at the freeze-line. In principle it is possible to eliminate e^*_{11}, e^*_{22}, etc., from the system of equations derived above and solve the resulting equations for a^*, h^* and v^* numerically. However, because of the complexity of the equations, e^*_{11}, e^*_{22}, etc., are retained as auxiliary dependent variables throughout the numerical integration. The equations of the reduced fourth-order system are set out in the appendix. They are arranged in a form ready for numerical integration by the Runge–Kutta procedure given the appropriate starting conditions. The integration was carried out from $X = X_z$ to $X = 0$, using boundary conditions given below. For each B, T_z and X_z, the solutions of the steady flow problem are pre-computed and read into the computer as they are required.

The actual strategy adopted to locate the eigencombination (B, T_z, X_z, Ω) is based on the scheme developed by Mack (1965) to deal with the stability of forced-flow compressible boundary layers. Briefly, for each set of the four parameters, the procedure involves finding three sets of linearly independent solutions of the fifth-order system satisfying the following linearly independent boundary conditions at $X = X_z$:

$$v^* = 0, \quad D\bar{A}a^*/DT = 0$$

and

$$\begin{aligned} &\text{set 1:} \quad a^* = 1, \quad dh^*/dX = 0, \quad h^* = 0;\\ &\text{set 2:} \quad a^* = 0, \quad dh^*/dX = 1, \quad h^* = 0;\\ &\text{set 3:} \quad a^* = 0, \quad dh^*/dX = 0, \quad h^* = 1.\end{aligned}$$

The first two boundary conditions are common to all three sets. It is clear that these sets of solutions span the space of all solutions of the fifth-order system which satisfy the boundary conditions (14) and (15). These solutions are linearly combined such that the first two boundary conditions of (13) are satisfied. In general the final boundary condition of (13) will not be satisfied. For a given set of B, T_z and X_z, a simple linear search procedure is used to vary Ω_r and Ω_i until the final boundary condition is met as closely as desired. This then gives an eigencombination (B, T_z, X_z, Ω). Using this procedure, points on either side of the neutral surface are obtained from which the neutral surface is located by interpolation.

Some results of the above computations are shown in figures 3(*a*)–(*c*). These are neutral-stability curves for $X_z = 7$, 8 and 10. On each curve $\Omega_i = 0$ and Ω_r (shown in parentheses) varies continuously. Some typical eigenvalues and the associated blow ratio R and reciprocal draw ratio $\bar{H}(X_z)$ are tabulated in table 1.

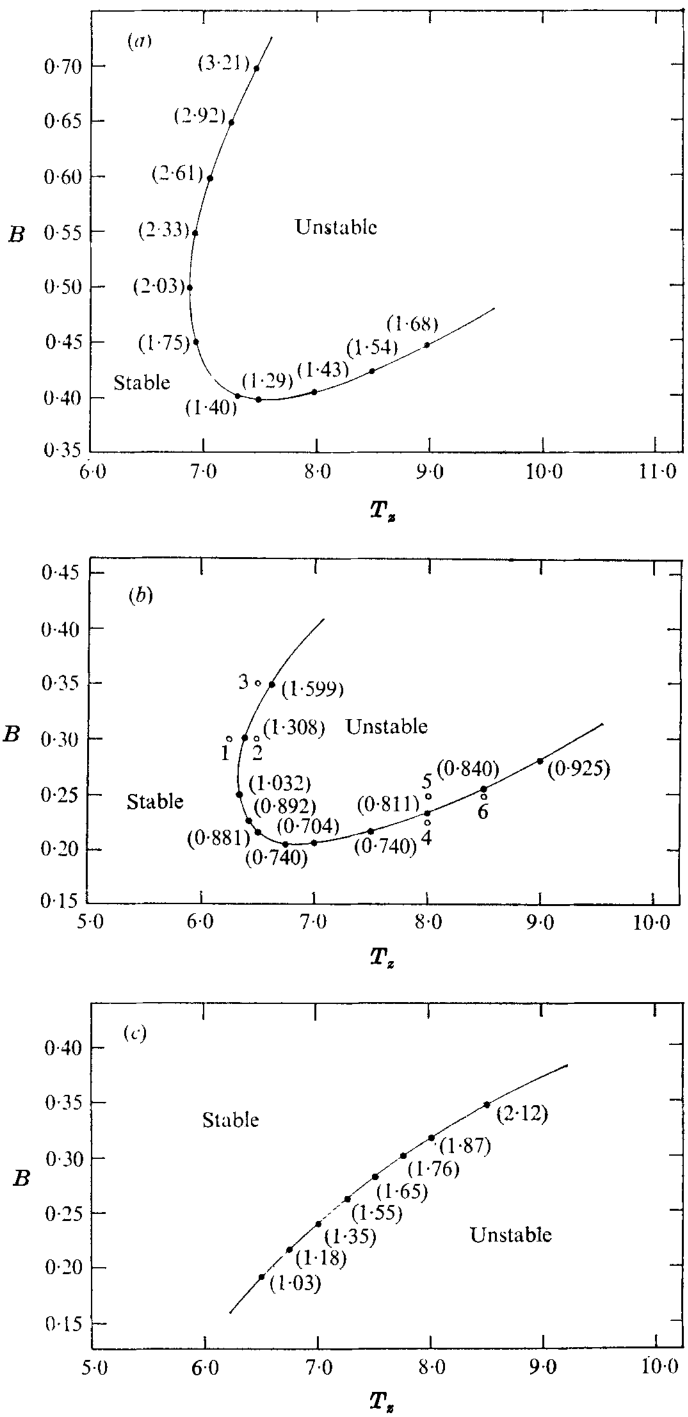


FIGURE 3. Neutral-stability curves for (*a*) $X_z = 7{\cdot}0$, (*b*) $X_z = 8{\cdot}0$ and (*c*) $X_z = 10{\cdot}0$. Values of Ω_r indicated on curves. In (*b*), the open circles correspond to points in table 1.

Point in figure 3(*b*)	B	T_z	Ω_r	Ω_i	R	$\bar{H}(X_z)$
1	0·30	6·25	1·3119	−0·0131	4·0798	$1{\cdot}18\times10^{-4}$
2	0·30	6·50	1·3056	+0·0130	4·1661	$7{\cdot}52\times10^{-5}$
3	0·35	6·50	1·5991	−0·0033	3·8222	$1{\cdot}08\times10^{-4}$
4	0·225	8·00	0·7894	−0·0910	5·1033	$7{\cdot}39\times10^{-7}$
5	0·25	8·00	0·8338	+0·0891	5·0154	$1{\cdot}57\times10^{-6}$
6	0·25	8·50	0·8048	−0·0283	4·9737	$3{\cdot}82\times10^{-7}$

TABLE 1. Typical eigenvalues for $X_z = 8{\cdot}00$

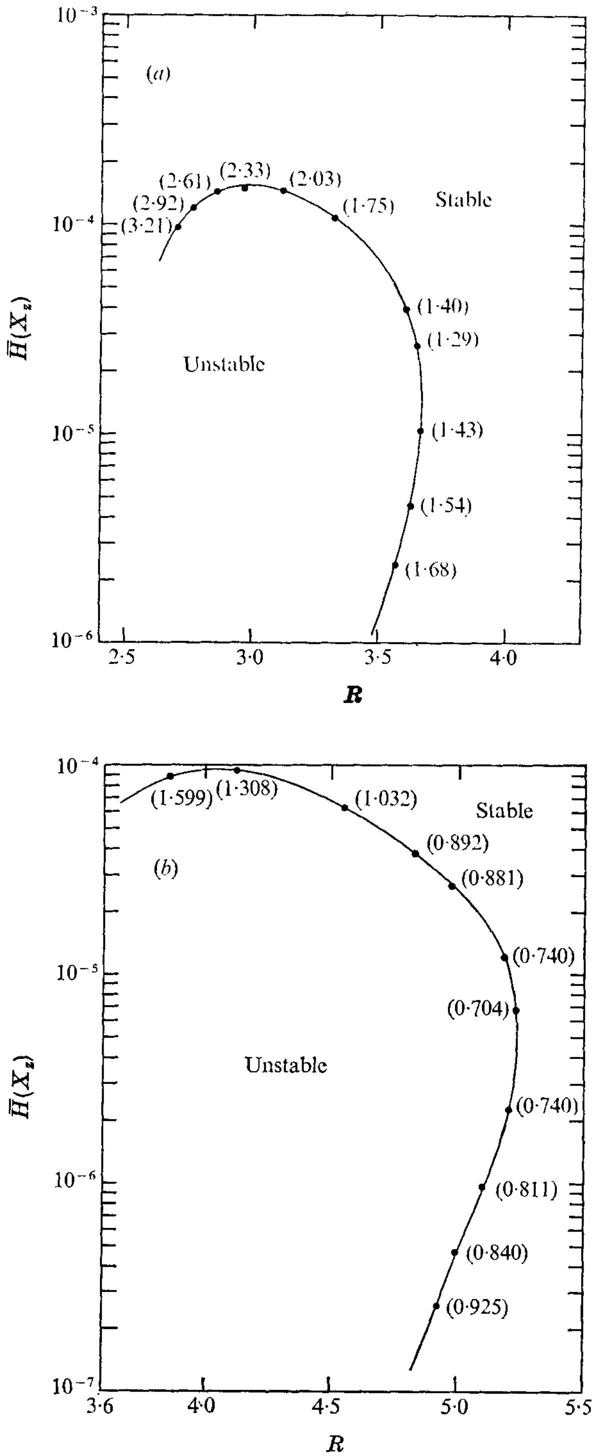
These correspond to points indicated on figure 3(*b*). The amount of computation involved in locating each neutral-stability curve with X_z held constant is large. This has prevented the construction of a more complete set of neutral-stability curves with $5 \leqslant X_z \leqslant 20$, particularly those with $X_z \geqslant 10$.

5. Discussion

From figures 3(*a*)–(*c*) it can be seen that the transition from stability to instability is in the general direction of increasing B and T_z. It is interesting to note that the ranges of values of B and T_z that are likely to be encountered in normal film-blowing operations lie in the stable region of the B, T_z plane. The draw ratios on the neutral-stability curves are of the order of 10^3–10^7 for the three X_z's considered, the larger values corresponding to the longer bubbles. Such draw ratios are much higher than those encountered in practice. The blow ratios on the neutral-stability curves vary from 3·0 to 6·5. These are slightly larger than the normal blow ratio observed in practice but still within the attainable range. Even from the limited results obtained it can be concluded that the film-blowing process, as modelled by the simple isothermal tubular film flow, is a stable process under normal operating conditions as far as axisymmetric disturbances are concerned. This is true for $X_z = 7$, 8 and 10. For $X_z \leqslant 6$ all attempts to find unstable solutions of the eigenvalue problem have failed. In the region of the B, T_z plane $0{\cdot}1 \leqslant B \leqslant 1{\cdot}0$ and $2{\cdot}0 \leqslant T_z \leqslant 12{\cdot}0$ only stable eigen-solutions were found.

The dimensionless parameter B represents the ratio of the pressure force to a typical viscous force while T_z is the ratio of the applied tension to the typical viscous force. On the upper arms of figures 3(*a*) and (*b*) increasing T_z is accompanied by a transition from stability to instability. Hence f_z is a destabilizing force there. However on the lower arms of figures 3(*a*) and (*b*) increasing f_z leads to stability. Hence f_z is a stabilizing force there. The role of Δp, assumed to be unaffected by the introduction of disturbances, is just the opposite. On the upper arms of the neutral-stability curves it is stabilizing and on the lower arms, destabilizing.

With the limited results it is not possible to study quantitatively the effect of freeze-line height X_z on the stability of the tubular film flow. However, from the general positions of the neutral-stability curves in figures 3(*a*)–(*c*) it appears



FIGURES 4 (*a*, *b*). For legend see facing page.

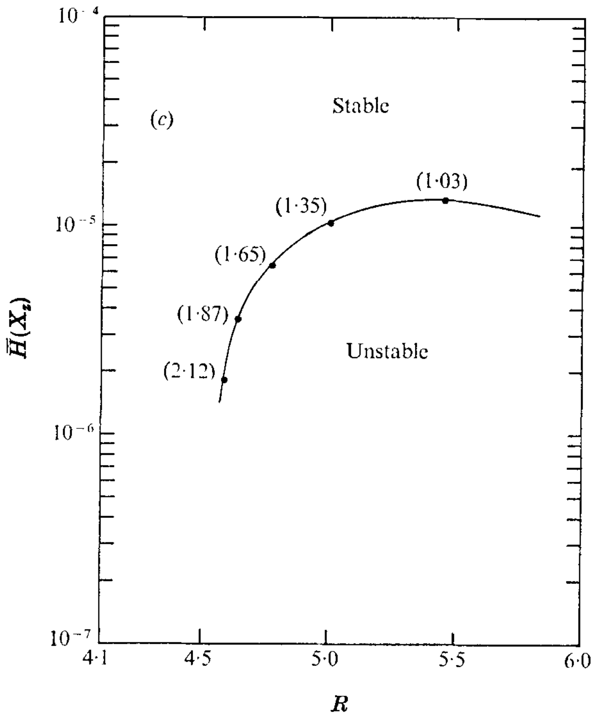


FIGURE 4. Neutral-stability curves for (a) $X_z = 7{\cdot}0$, (b) $X_z = 8{\cdot}0$ and (c) $X_z = 10{\cdot}0$ in the R, $\bar{H}(X_z)$ plane.

that as X_z is increased the neutral-stability curve moves nearer to the region of the B, T_z plane encountered in noımal film-blowing operation. In this sense it can be said that short bubbles are more stable to axisymmetric disturbances than long bubbles.

The blow ratio and draw ratio vary continuously on a neutral-stability curve in the B, T_z plane. The relationship between R and $\bar{H}(X_z)$ on the neutral-stability curves is of special interest too perators of the film-blowing process since in normal operation R and $\bar{H}(X_z)$ are specified rather than B and T_z. The neutral-stability curves in figure 3 are replotted in the R, $\bar{H}(X_z)$ plane in figure 4. X_z is again held fixed on each of the curves. Values of Ω_r are shown in parentheses. The stable and unstable regions of the R, $\bar{H}(X_z)$ plane are as indicated on the figure. Again the limited data available do not permit a systematic study of the effects of changing R and $\bar{H}(X_z)$ on the stability of the bubble.

The stability of isothermal tubular film flow as such is probably not directly relevant to the actual operation of the film-blowing process. However the general procedure developed in the formulation of the eigenvalue problem can be extended to deal with the stability of more realistic models of the film-blowing operation. Many factors, e.g. effects of temperature variation within the flow field analysed, deviations from Newtonian behaviour, and effects of gravity and inertia, have been neglected in the present simple model of the process; these have been discussed by Pearson & Petrie (1970*b*, *c*) and possible improvements suggested. In a recent paper Petrie (1975) extended the analysis to include many of these neglected factors and obtained the bubble shape $\bar{A}(X)$ and film thickness $\bar{H}(X)$ for the improved models. It is worth noting that, while the steps leading to the eigenvalue problems for these improved models of Petrie will remain basically the same as that of the present analysis, the resulting problems will almost certainly be much more complicated. More efficient numerical and computational schemes will have to be devised to deal with these problems.

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Appendix. Equations for axisymmetric disturbances

$$da^*/dX = d^*, \tag{A 1}$$

$$d(Aa^*)/dX = a^*\,dA/dX + Ad^*, \tag{A 2}$$

$$e_{33}^* = -i\Omega a^* + E_{33}\left[v_1^* - \frac{d(Aa^*)}{dX}\cos\theta\sin\theta\right] + V_1\cos\theta\, d^*, \tag{A 3}$$

$$t_{11}^* = \{[T_{11}HA\cos\theta(h^* + a^* - \cos\theta\sin\theta\, d(Aa^*)/dX - A^2a^*]_X^{X_z} + (HA\cos\theta\, t_{11}^*)_{X_z}\}/(HA\cos\theta)_X, \tag{A 4}$$

$$e_{22}^* = -\tfrac{1}{2}(e_{33}^* + Bt_{11}^*), \tag{A 5}$$

$$t_{33}^* = (e_{33}^* - e_{22}^*)/B, \tag{A 6}$$

$$\frac{dh^*}{dX} = \left(i\Omega h^* - E_{22}\left(v_1^* - \cos\theta\sin\theta\frac{d(Aa^*)}{dX} - h^*\right) + e_{22}^*\right)\Big/(V_1\cos\theta) - \frac{dH}{dX}h^*/H, \tag{A 7}$$

$$r_3^*/R_3 = a^* + \cos\theta\sin\theta\, d(Aa^*)/dX, \tag{A 8}$$

$$r_1^*/R_1^2 = (t_{11}^*/T_{11} + h^*)/R_1 + (h^* - r_3^*/R_3)\,T_{33}/(T_{11}R_3) + t_{33}^*/(T_{11}R_3), \tag{A 9}$$

$$e_{11}^* = -(e_{22}^* + e_{33}^*), \tag{A 10}$$

$$\frac{dv_1^*}{dX} = \left((i\Omega + E_{11})\sin\theta\frac{d(Aa^*)}{dX} + e_{11}^*\sec\theta - \frac{dV_1}{dX}v_1^*\right)\Big/V_1, \tag{A 11}$$

$$\frac{dd^*}{dX} = \left(\left(r_1^*/R_1^2 - 3\sin\theta\cos\theta\frac{d(Aa^*)}{dX}\Big/R_1\right)\sec^3\theta - 2\frac{dA}{dX}\frac{da^*}{dX} - \frac{d^2A}{dX^2}a^*\right)\Big/A. \tag{A 12}$$

These equations are obtained as follows.

(A 1), (A 2): from definition of d^*.
(A 3): from (7).
(A 4): rearrangement of (12).
(A 5): combination of (8) and (9).

(A 6): from (8).
(A 7): rearrangement of (7).
(A 8): rearrangement of (11).
(A 9): rearrangement of (10).
(A 10): from (9).
(A 11): rearrangement of (7).
(A 12): rearrangement of (11).

REFERENCES

ARIS, R. 1962 *Vectors, Tensors, and the Basic Equations of Fluid Mechanics*. Prentice Hall.

MACK, L. M. 1965 In *Methods in Computational Physics*, vol. 4 (ed. B. Alder *et al.*), p. 247.

PEARSON, J. R. A. & PETRIE, C. J. S. 1970*a* *J. Fluid Mech.* **40**, 1.

PEARSON, J. R. A. & PETRIE, C. J. S. 1970*b* *Plast. Poly.* **38**, 85.

PEARSON, J. R. A. & PETRIE, C. J. S. 1970*c* *J. Fluid Mech.* **42**, 609.

PETRIE, C. J. S. 1975 *A.I.Ch.E. J.* **21**, 275.

SCHENKEL, G. 1966 *Plastics Extrusion Technology*. London: Iliffe.

YEOW, Y. L. 1972 Ph.D. thesis, University of Cambridge.